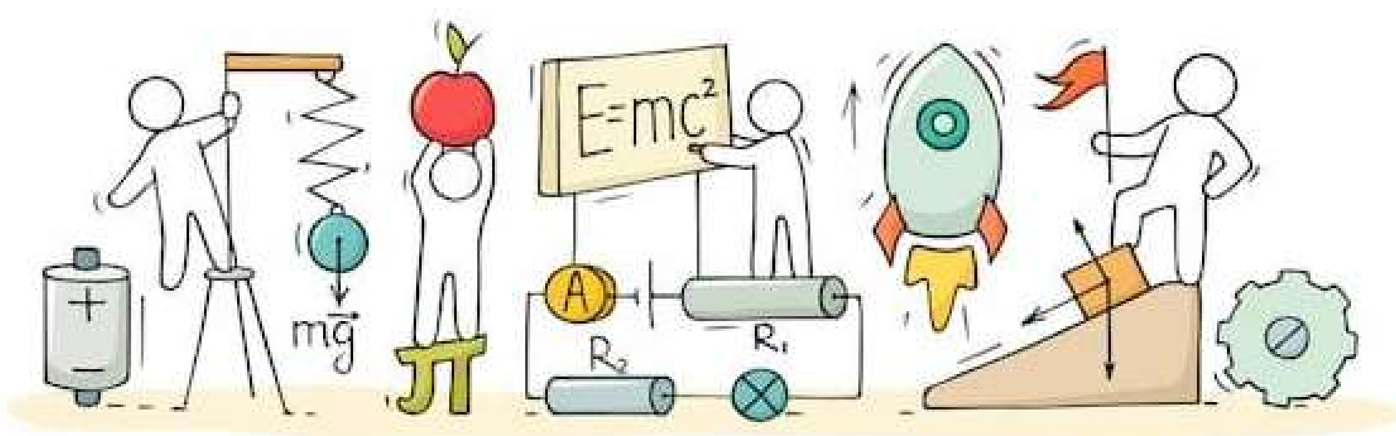
PHYSICS

Chapter 7: System of Particles and Rotational Motion



System of Particles and Rotational Motion

Top Formulae

Position vector of COM of a system	$\vec{R} = \dfrac{m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r_3} + ...}{m_1 + m_2 + m_3 + ...}$
Coordinates of COM	$x = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + ...}{m_1 + m_2 + m_3 + ...}$ $y = \dfrac{m_1y_1 + m_2y_2 + m_3y_3 + ...}{m_1 + m_2 + m_3 + ...}$ $z = \dfrac{m_1z_1 + m_2z_2 + m_3z_3 + ...}{m_1 + m_2 + m_3 + ...}$
Velocity of COM of a system of two particles	$\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_1}{m_1 + m_2}$
Equations of rotational motion	i) $\omega = \omega_1 \quad \alpha t$ ii) $\theta = \omega_1 t + \dfrac{1}{2}\alpha t^2$ iii) $\omega_2 - \omega^2 = 2\alpha\theta$
Centripetal acceleration	$= \dfrac{v^2}{r} = r\omega^2$
Linear acceleration	$a = r\alpha$
Angular momentum	$\vec{L} = \vec{r} \times \vec{p}$
Torque	$\vec{\tau} = \vec{r} \times \vec{F}$
Kinetic energy of rotation	$= \dfrac{1}{2}I\omega^2$
Kinetic energy of translation	$= \dfrac{1}{2}mv^2$
Total kinetic energy	$= \dfrac{1}{2}I\omega^2 + \dfrac{1}{2}mv^2$
Angular momentum	$L = I\omega$
Torque	$\tau = I\alpha$
Relation between torque and angular momentum	$\vec{\tau} = \dfrac{d\vec{L}}{dt}$
Moment of inertia in terms of radius of gyration	$I = \sum_{i=1}^{i=n} m_i r_i^2 = MK^2$

Moment of inertia of a uniform circular ring about an axis passing through the centre and perpendicular to the plane of the ring	$I = MR^2$
For a uniform circular disc	$I = \frac{1}{2}MR^2$
For a thin uniform rod	$I = \frac{1}{12}M\ell^2$
For a hollow cylinder about its axis	$I = MR^2$
For a solid cylinder about its axis	$I = \frac{1}{2}MR^2$
For a hollow sphere about its diameter	$I = \frac{2}{3}MR^2$
For a solid sphere about its diameter	$I = \frac{2}{5}MR^2$
Coefficient of friction for rolling of solid cylinder without slipping down the rough inclined plane	$\mu = \frac{1}{3}\tan\theta$

Top Concepts

- A rigid body is a solid body of finite size in which deformation is negligible under the effect of deforming forces.
- A rigid body is one for which the distances between different particles of the body do not change.
- The centre of mass (COM) of a rigid body is the point in or near an object at which the whole mass of the object may be considered to be concentrated.
- A rigid object can be substituted with a single particle with mass equal to the total mass of the system located at the COM of the rigid object.
- In pure translational motion, all particles of the body move with the same velocities in the same direction.
- In pure translational motion, every particle of the body moves with the same velocity at any instant of time.
- In rotational motion, each particle of the body moves along the circular path in a plane perpendicular to the axis of rotation.

- In rotation about a fixed axis, every particle of the rigid body moves in a circle with same angular velocity at any instant of time.
- Irrespective of where the object is struck, the COM always has translational motion.
- Motion of the COM is the resultant of the motions of all the constituent particles of a system.
- Velocity of the centre of mass of a system of particles is given by

$$\vec{V} = \frac{\vec{P}}{M}$$

where $\vec{P}$ is the linear momentum of the system.

- The translational motion of the centre of mass of a system is as if all the mass of the system is concentrated at this point and all the external forces act at this point.

If the net external force on the system is zero, then the total linear momentum of the system is constant and the centre of mass moves at a constant velocity.

- Torque is the rotational analogue of force in translational motion.
- The torque or moment of force on a system of n particles about the origin is the cross product of radius vectors and force acting on the particles.

$$\vec{\tau} = \sum_{i=1}^{n} \vec{r} \times \vec{F}_i$$

- Angular velocity in rotational motion is analogous to linear velocity in linear motion.
- Conditions for equilibrium:
- The resultant of all the external forces must be zero. The resultant of all the external torques must be zero.
- The centre of gravity is the location in the extended body where we can assume the whole weight of the body to be concentrated.
- When a body acted upon by gravity is supported or balanced at a single point, the centre of gravity is always at and directly above or below the point of suspension.
- The moment of inertia of a rigid body about an axis is defined by the formula $I = \sum m_i r_i^2$, where r_i is the perpendicular distance of the i^{th} point of the body from the axis. The kinetic energy of rotation is

$$K = \frac{1}{2}I\omega^2$$

- **Theorem of perpendicular axis:** The moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with the perpendicular axis and lying in the plane of the body.

  $I = I_x + I_y$

  Here

  I_x: Moment of inertia about the x axis in the plane of the lamina.

  I_y: Moment of inertia about the y axis in the plane of the lamina.

- **Theorem of parallel axes:** The moment of inertia of a body about any axis is equal to its moment of inertia I_{cm} about a parallel axis through its centre of mass plus the product of the mass M of the body and the square of the distance between the two axes.

  $I_p = I_{cm} + Md^2$

- Work done on a rigid body by the external torque is equal to the change in its kinetic energy.

- Pure rolling implies rolling without slipping. It occurs when there is no relative motion at the point of contact where the rolling object touches the ground.

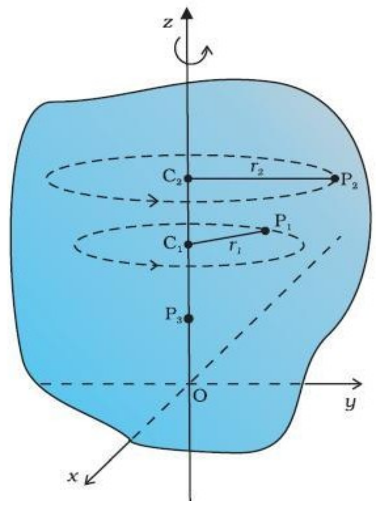
- For a rolling wheel of radius r which is accelerating, the acceleration of the centre of mass is $a_{cm} = R\alpha$

- Law of conservation of angular momentum: If the net resultant external torque acting on an isolated system is zero, then total angular momentum L of the system should be conserved.

- The relation between the arc length S covered by a particle on a rotating rigid body at a distance r from the axis and the displacement θ in radians is given by $S = r\theta$.

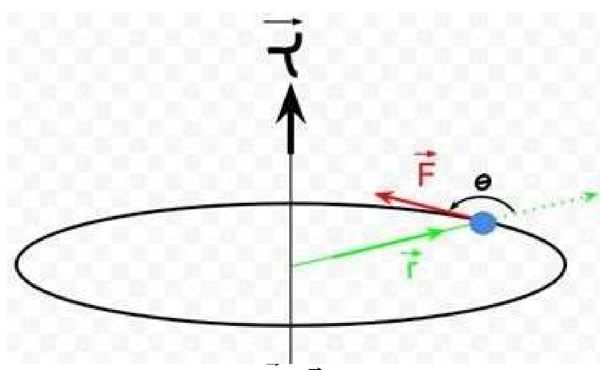
Diagrams

Rigid body in motion about a fixed axis



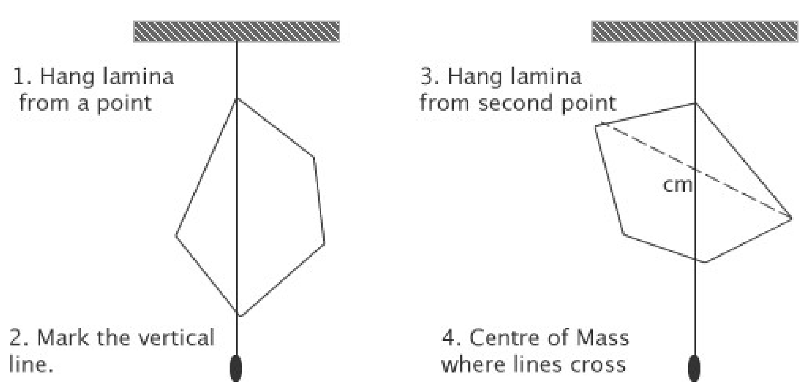
A rigid body rotation about the z-axis (Each point of the body such as P_1 or P_2 describes a circle with its centre (C_1 or C_2) on the axis. The radius of the circle (r_1 or r_2) is the perpendicular distance of the point (P_1 or P_2) from the axis. A point on the axis like P_3 remains stationary).

Moment of force

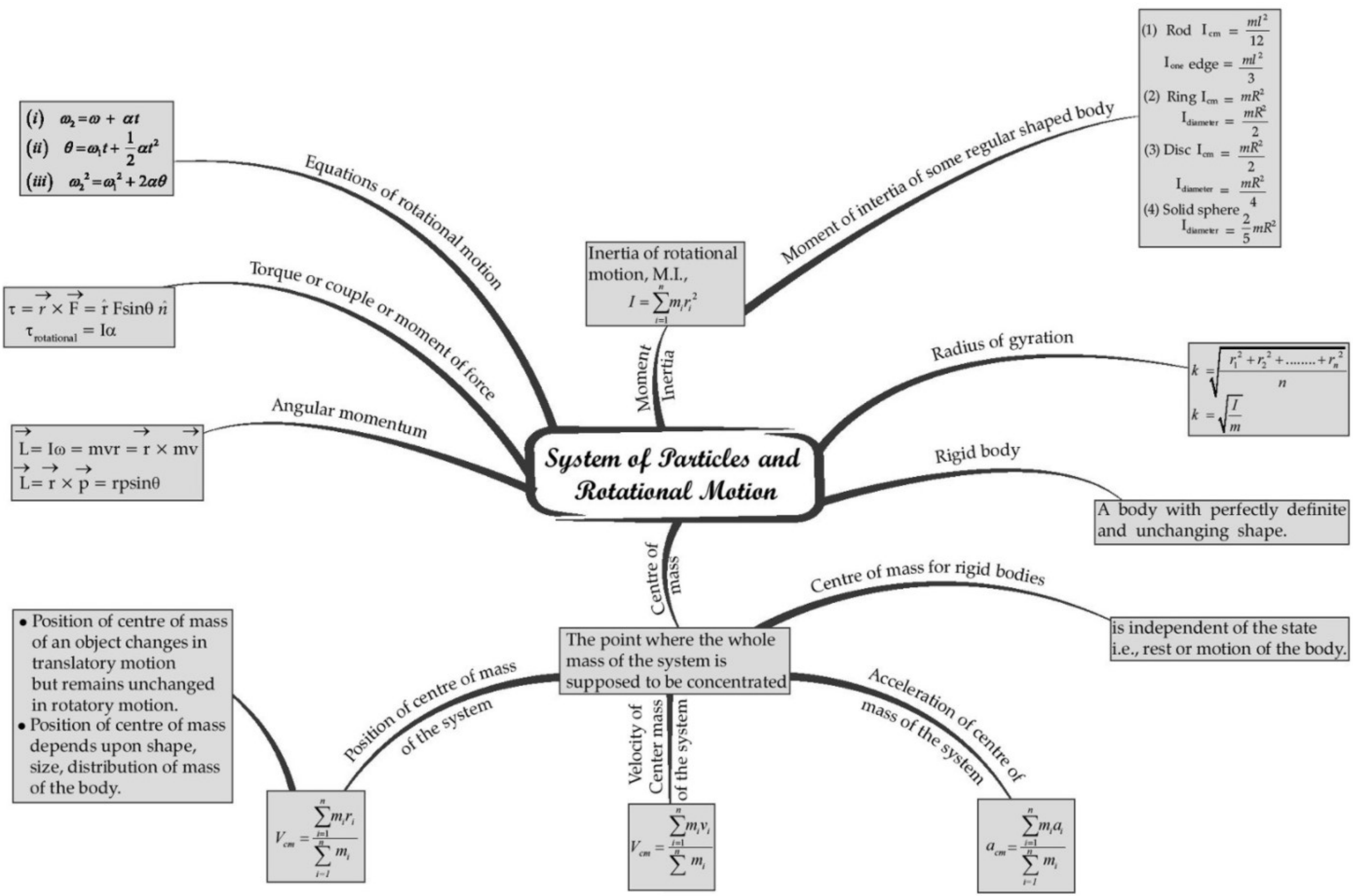


$\vec{\tau} = \vec{F} \times \vec{r}$

Centre of gravity of an irregularly shaped body



CHAPTER - 7 : SYSTEM OF PARTICLES AND ROTATIONAL MOTION



Important Questions

Multiple Choice questions-

1. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is

(a) L/2

(b) L/4

(c) 2 L

(d) 4 L

2. A car is moving with a speed of 108 km/hr on a circular path of radius 500 m. Its speed is increasing at the rate of 2 m/s. What is the acceleration of the car?

(a) 9.8 m/s^2

(b) 2.7 m/s^2

(c) 3.6 m/s^2

(d) 1.8 m/s^2

3. The moment of inertia of uniform circular disc about an axis passing its center is 6kgm^2. its M.I. about an axis perpendicular to its plane and just touching the rim will be

(a) 18 kg m^2

(b) 30 kg m^2

(c) 15 kg m^2

(d) 3 kg m^2

4. A particle undergoes uniform circular motion. About which point on the plane of the circle will the angular momentum of the particle remain conserved?

(a) center of the circle

(b) on the circumference of the circle

(c) inside the circle

(d) outside the circle

5. Two particles A and B, initially at rest, moves towards each other under a mutual force of attraction. At the instant when the speed of A is u and the speed of B is 2 u, the speed of center of mass is,

(a) Zero

(b) u

(c) 1.5 u

(d) 3 u

6. The moment of inertia of a body about a given axis is 1.2 kg metre^2. Initially, the body is at rest. In order to produce a rotating kinetic energy of 1500 joules, an angular acceleration of 25 radian/sec^2 must be applied about that axis for a duration of

(a) 4 sec

(b) 2 sec

(c) 8 sec

(d) 10 sec

7. Two discs has same mass rotates about the same axes. r1 and r2 are densities of two bodies (r1 > r2) then what is the relation between l1 and

(a) l2.

(b) l1 > l2

(c) l1 < l2

(d) l1 = l2

None of these

8. The kinetic energy of a body is 4 joule, and its moment of inertia is 2 kg m^2 then angular momentum is

(a) 4 kg m^2/sec

(b) 5 kg m^2/sec

(c) 6 kg m^2/sec

(d) 7 kg m^2/sec

9. A mass is revolving in a circle which is in the plane of the paper. The direction of angular acceleration is

(a) Upward to the radius

(b) Towards the radius

(c) Tangential

(d) At right angle to angular velocity

10. By keeping moment of inertia of a body constant, if we double the time period, then angular momentum of body

(a) Remains constant

(b) Becomes half

(c) Doubles

(d) Quadruples

Very Short Question:

1. Can the geometrical centre and C.M. of a body coincide? Give examples.
2. How does the M.I. change with the speed of rotation?
3. Under what conditions, the torque due to an applied force is zero?
4. Is it correct to say that the C.M. of a system of n-particles is always given by average position vectors of the constituent particles? If not, when the statement is true?
5. A cat is able to land on her feet after a fall. Which principle of Physics is being used by her?
6. What is conserved when a planet revolves around a star?
7. If no external torque acts on a body, will its angular velocity remain conserved?
8. A body is rotating at a steady rate. Is a torque acting on the body?
9. What is the other name for angular momentum?
10. Out of two spheres of equal masses, one rolls down a smooth inclined plane of height h and the other is falling freely through height h. In which case, the work done is more?

Short Questions:

1. What is the difference between the centre of gravity and C.M.?
2. There are two spheres of the same mass and radius, one is solid, and the other is hollow. Which of them has a larger moment of inertia about its diameter?
3. What shall be the effect on the length of the day if the polar ice caps of Earth melt?
4. If only an external force can change the state of motion of the C.M. of a body, how does it happen that the internal force of brakes can bring a vehicle to rest?
5. What do you understand by a rigid body?
6. What do you understand by a rigid body?
7. Two equal and opposite forces act on a rigid body. Under what conditions will the body (a) rotate, (Z>) not rotate?
8. (a) Why is it easier to balance a bicycle in motion?

   (b) Why spokes are fitted in the cycle wheel?

Long Questions:

1. Discuss the rolling of a cylinder (without slipping) down a rough inclined plane and obtain an expression for the necessary coefficient of friction between the cylinder and the surface.

2. Prove that

   (a) $\Delta\omega = \tau \Delta\theta$

   (b) $P = \tau \omega$.

Assertion Reason Questions:

1. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
   (c) Assertion is correct, reason is incorrect
   (d) Assertion is incorrect, reason is correct.

   Assertion: The Centre of mass of a body may lie where there is no mass.
   Reason: Centre of mass of body is a point, where the whole mass of the body is supposed to be concentrated.

2. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
   (c) Assertion is correct, reason is incorrect
   (d) Assertion is incorrect, reason is correct.

   Assertion: The earth is slowing down and as a result the moon is coming nearer to it.
   Reason: The angular momentum of the earth moon system is conserved.

Case Study Questions:

1. The cross product of two vectors is given by Vector C = A × B. The magnitude of the vector defined from cross product of two vectors is equal to product of magnitudes of the vectors and sine of angle between the vectors. Direction of the vectors is given by right hand corkscrew rule and is perpendicular to the plane containing the vectors.

∴ |vector C| = AB sin θ and Vector C = AB sin θ n

Where, cap n is the unit vector perpendicular to the plane containing the vectors A and B. Following are properties of vector product

a) Cross product does not obey commutative law. But its magnitude obeys commutative low.

$$\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A} \Rightarrow (\vec{A}\times\vec{B})$$
$$= -(\vec{B}\times\vec{A}),\ |\vec{A}\times\vec{B}|=|\vec{B}\times\vec{A}|$$

c) It obeys distributive law

$$\vec{A}\times(\vec{B}\times\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$$

d) The magnitude cross product of two vectors which are parallel is zero. Since θ = 0.

vector |A x B| = AB sin 0° = 0

e) For perpendicular vectors, θ = 90°, vector |A x B| = AB sin 90° |cap n| = AB

$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

$\hat{i} \times \hat{j} = \hat{k}$; $\hat{j} \times \hat{k} = \hat{i}$; $\hat{k} \times \hat{i} = \hat{j}$

$\hat{j} \times \hat{i} = -(\hat{i} \times \hat{j}) = -\hat{k}$; $\hat{k} \times \hat{j} = -(\hat{j} \times \hat{k}) = -\hat{i}$; $\hat{i} \times \hat{k} = -(\hat{k} \times \hat{i}) = -\hat{j}$

f) The expression for a × b can be put in a determinant form which is easy to remember

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

i. If θ is angle between two vectors, then resultant vector is maximum when θ is
   - a) 0
   - b) 90
   - c) 180
   - d) None of these

ii. Cross product is operation performed between
   - a) Two scalar numbers
   - b) One scalar other vector
   - c) 2 vectors
   - d) None of these

iii. Define cross product of two vectors

iv. State right hand screw rule for finding out direction of resultant after cross product of two vectors.

v. Give properties of cross product of parallel vector.

2. Radius of gyration: The radius of gyration of a body about an axis may be defined as the distance from the axis of a mass point whose mass is equal to the mass of the whole body and whose moment of inertia is equal to the moment of inertia of the body about the axis.

   the moment of inertia of a rigid body analogous to mass in linear motion and depends on the mass of the body, its shape and size, distribution of mass about the axis of rotation, and the position and orientation of the axis of rotation.

   Theorem of perpendicular axes

   It states that the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body. If we consider a planar body, An axis perpendicular to the body through a point O is taken as the z-axis. Two mutually perpendicular axes lying in the plane of the body and concurrent with z-axis, i.e., passing through O, are taken as the x and y-axes. The theorem states that

   $I z = I x + I y$.

   Theorem of parallel axes

   The moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes.

   z and z’ are two parallel axes, separated by a distance a. The z-axis passes through the centre of mass O of the rigid body. Then according to the theorem of parallel axes

   $I_{z'} = I_z + Ma^2$

   Where I_z and I_z' are the moments of inertia of the body about the z and z¢ axes respectively, M is the total mass of the body and a is the perpendicular distance between the two parallel axes.

   i. SI unit of radius of gyration

      a) Metre (m)

      b) M^2

      c) M^3

      d) None of these

   ii. Moment of inertia is analogous to

      a) Mass

      b) Area

      c) Force

      d) None of these

iii. Define radius of gyration

iv. State Theorem of perpendicular axes

v. State Theorem of parallel axes

✓ Answer Key:

Multiple Choice Answers-

1. Answer: (b) L/4
2. Answer: (b) 2.7 m/s^2
3. Answer: (a) 18 kg m^2
4. Answer: (a) center of the circle
5. Answer: (a) Zero
6. Answer: (b) 2 sec
7. Answer: (b) $I1 > I2$
8. Answer: (a) 4 kg m^2/sec
9. Answer: (c) Tangential
10. Answer: (b) Becomes half

Very Short Answers:

1. Answer: Yes, C.M. and geometrical centre may coincide when the body has a uniform mass density, e.g. C.M. and geometrical centre are the same in case of a sphere, cube and cylinder etc.
2. Answer: M.I. is not affected by the speed of rotation of the body.
3. Answer:

   We know that $\tau = rF \sin \theta$. If $\theta = 0$ or 180,

   or

   $r = 0$, then $\tau = 0$, $r = 0$ means the applied force passes through the axis of rotation.
4. Answer: No, this statement is true when all the particles of the system are of the same mass.
5. Answer: Principle of conservation of angular momentum.
6. Answer: Angular momentum.
7. Answer: No, it is the angular momentum that will be conserved.
8. Answer: No, torque is required only for producing angular acceleration.
9. Answer: Moment of momentum.

10.Answer: Moment of momentum.

Short Questions Answers:

1. Answer: C.G.: It is the point where the whole of the weight of the body is supposed to be concentrated i.e. on this point, the resultant of the gravitational force on all the particles of the body acts.

   C.M.: It is the point where the whole of the mass of the body may be supposed to be concentrated to describe its motion as a particle.

2. Answer: The hollow sphere shall have greater M.I., as its entire mass is concentrated at the boundary of the sphere which is at maximum distance from the axis.
3. Answer: Melting of polar ice caps will produce water spread around the Earth going farther away from the axis of rotation that will increase the radius of gyration and hence M.I. In order to conserve angular momentum, the angular velocity ω shall decrease. So the length of the day $\left(T = \frac{2\pi}{\omega}\right)$ shall increase.
4. Answer: The internal force of brakes converts the rolling friction into sliding friction. When brakes are applied, wheels stop rotating. When they slide, the force of friction comes into play and stops the vehicle. It is an external force.
5. Answer: A rigid body is that in which the distance between all the constituting particles remains fixed under the influence of external force. A rigid body thus conserves its shape during its motion.
6. Answer:
   - The mutual forces between the particles of a system are called internal forces.
   - The forces exerted by some external source on the particles of the system are called external forces.
7. Answer: Two equal and opposite forces acting on a rigid body such that their lines of action don't coincide constitute a couple. This couple produces a turning effect on the body. Hence the rigid body will rotate. If the two equal and opposite forces act in such a way that their lines of action coincide, then the body will not rotate.
8. Answer:

   (a) The rotating wheels of a bicycle possess angular momentum. In the absence of an external torque, neither the magnitude nor the direction of angular momentum can change. The direction of angular momentum is along the axis of the wheel. So the bicycle does not get tilted.

   (b) The cycle wheel ¡s constructed in such a way so as to increase the M.I. of the wheel with minimum possible mass, which can be achieved by using spokes and the M.I. is increased to ensure the uniform speed.

Long Questions Answers:

1. Answer:

Consider a solid cylinder of mass m, radius R and MJ. I rolling down an inclined plane without slipping as shown in the figure. The condition of rolling down without slipping means that at each instant of time, the point of contact P of the cylinder with the inclined plane is momentarily at rest and the cylinder is rotating about that as the axis.

Let θ = angle of inclination of the plane. The forces acting on the cylinder are:

- The weight mg of the cylinder acting vertically downward.
- The force of friction F between the cylinder and the surface of the inclined plane and acts opposite to the direction of motion.
- The normal reaction N due to the inclined plane acting normally to the plane at the point of contact. The weight W of the cylinder can be resolved into two rectangular components:

  (a) mg cos θ along ⊥ to the inclined plane.

  (b) mg sin θ along the inclined plane and in the downward direction. It makes the body move downward.

Let a = linear acceleration produced in the cylinder,

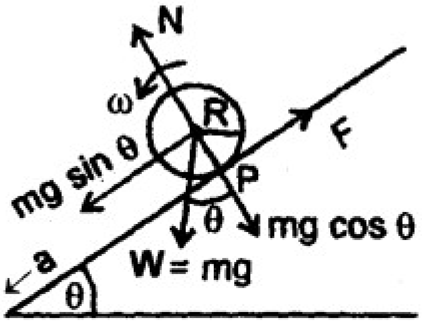
Then according to Newton’s 2nd law of motion,

$ma = mg \sin \theta - F$ .... (1)

and $N = mg \cos \theta$ .... (2)

If α = angular acceleration of the cylinder about the axis of rotation, then

$\tau = I \alpha$ .... (3)



Here, τ is provided by F i.e.

$\tau = F.R$ .... (4)

∴ from (3) and (4), we get

$$I\alpha = FR$$

or $$F = \frac{I\alpha}{R} = \frac{I}{R}.\frac{a}{R} \qquad (\because a = R\alpha)$$

$$= \frac{Ia}{R^2} \qquad .... (5)$$

$\therefore$ from (1) and (5), we get

$$ma = mg\sin\theta - \frac{Ia}{R^2}$$

or $$ma + \frac{Ia}{R^2} = mg\sin\theta$$

or $$a\left(m + \frac{I}{R^2}\right) = mg\sin\theta$$

or $$a = \frac{mg\sin\theta}{m\left(1 + \frac{I}{mR^2}\right)}$$

or $$a = \frac{g\sin\theta}{1 + \frac{I}{mR^2}} \qquad .... (6)$$

For solid cylinder, $$I = \frac{1}{2}mR^2$$

$\therefore$ $$a = \frac{g\sin\theta}{\left(1 + \frac{1}{2}\right)} = \frac{2}{3}g\sin\theta$$

$\therefore$ From (5) and (6), we get

$$F = \frac{I}{R^2}\cdot\frac{g\sin\theta}{\left(1 + \frac{I}{mR^2}\right)}$$

$$F = \frac{mg\sin\theta}{\left(1 + \frac{mR^2}{I}\right)} \qquad .... (7)$$

$$= \frac{mg\sin\theta}{(1 + 2)} \qquad \left(\therefore I = \frac{1}{2}mR^2\right)$$

$$F = \frac{1}{3}mg\sin\theta \qquad .... (8)$$

If μ_s be the coefficient of static friction between the cylinder and the surface,

Then

$$\mu_s = \frac{F}{N} = \frac{\frac{1}{3} mg \sin\theta}{mg\cos\theta}$$

$$= \frac{1}{3} \tan\theta$$

For rolling without slipping

$$\frac{F}{N} \leq \mu_s$$

or $$\frac{1}{3} \tan\theta < \mu_s \quad \text{.... (9)}$$

equation (9) is the required condition for rolling without slipping i.e., $\frac{1}{3}$ tan θ should be less than equal to μs i.e., the maximum allowed inclination of the plane with the horizontal is given by

$\theta_{max} = \tan^{-1} (3\ \mu_s)$

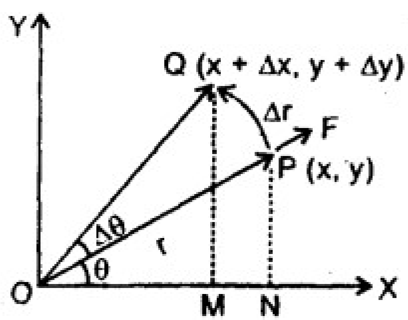
2. Answer:

(a) $\Delta\omega = \tau\ \Delta\theta$

Let F = force applied on a body moving in XY plane.

Δr = linear displacement produced in the body by the force F in moving from P to Q.

If Δω is the small work done by the force, then by definition of work.



ΔW = F . Δr ....(1)

In component form,

$$\mathbf{F} = F_x\hat{i} + F_y\hat{j}$$

and $$\Delta\mathbf{r} = \Delta x\hat{i} + \Delta y\hat{j} \quad \text{.... (2)}$$

$\therefore$ from (1) and (2), we get

$$\Delta W = (F_x\hat{i} + F_y\hat{j}).(\Delta x\hat{i} + \Delta y\hat{j})$$
$$= F_x\Delta x + F_y\Delta y \quad \text{.... (3)}$$

Let PN ⊥ on X-axis & PON = θ

∴ in rt ∠d ΔPNO,

$$\sin\theta = \frac{y}{r} \quad \text{.... (4)}$$

and $$\cos\theta = \frac{x}{r} \quad \text{.... (5)}$$

Also in ΔQMO,

$$x + \Delta x = r\cos(\theta + \Delta\theta)$$

and $$y + \Delta y = r\sin(\theta + \Delta\theta)$$

As Δθ is very small, i.e. Δθ → 0, cos Δθ → 1 and sin Δθ → Δθ

$$\therefore \quad x + \Delta x = r(\cos\theta\cos\Delta\theta - \sin\theta\sin\Delta\theta)$$
$$= r(\cos\theta \,.\, 1 - \sin\theta \,.\, \Delta\theta)$$
$$= x - y\,\Delta\theta$$

or $$\Delta x = -y\,\Delta\theta \quad \text{.... (6)}$$

and $$y + \Delta y = r(\sin\theta\cos\Delta\theta + \cos\theta\sin\Delta\theta)$$
$$= r(\sin\theta \,.\, 1 + \cos\theta \,.\, \Delta\theta)$$
$$= y + x\,\Delta\theta$$

or $$\Delta y = x\,\Delta\theta \quad \text{.... (7)}$$

$\therefore$ from (3), (6) and (7), we get

$$\Delta\omega = F_x(-y\,\Delta\theta) + F_y(x\,\Delta\theta)$$
$$= (x\,F_y - y\,F_x)\,\Delta\theta$$
$$= \tau\,\Delta\,\theta$$

(b) P = τ ω.

We know that $$P = \frac{\Delta\omega}{\Delta t} = \frac{\tau\Delta\theta}{\Delta t} = \tau\frac{\Delta\theta}{\Delta t} = \tau\omega$$

where $$\frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \omega \text{ if } \Delta t \to 0$$

$$\therefore \quad P = \tau\omega.$$

Assertion Reason Answer:

1. (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

   Explanation

   As the concept of Centre of mass is only theoretical, therefore in practice no mass may lie at the Centre of mass. For example, Centre of mass of a uniform circular ring is at the Centre of the ring where there is no mass.

2. (d) The earth is not slowing down. The angular momentum of the earth – moon system is conserved.

   Explanation:

   The earth is not slowing down. The angular momentum of the earth – moon system is conserved.

Case Study Answer:

1. **Answer**

   i. (a) 0

   ii. (c) 2 vectors

   iii. The cross product of two vectors is given by Vector C = A × B. The magnitude of the vector defined from cross product of two vectors is equal to product of magnitudes of the vectors and sine of angle between the vectors.
   ∴ |vector C| = ABsinθ and Vector C = ABsinθ n. Where, cap n is the unit vector perpendicular to the plane containing the vectors A and B.

   iv. We can find the direction of the unit vector with the help of the right-hand rule. In this rule, we can stretch our right hand so that the index finger of the right hand in the direction of the first vector and the middle finger is in the direction of the second vector. Then, the thumb of the right hand indicates the direction or unit vector n.

   v. The cross product of two vectors is zero vectors if both the vectors are parallel or opposite to each other. Conversely, if two vectors are parallel or opposite to each other, then their product is a zero vector. Two vectors have the same sense of direction. $\theta = 90^0$ As we know, sin 0° = 0 and sin 90° = 1

$$\vec{X} \times \vec{Y} = |\vec{X}|.|\vec{Y}|sin\theta$$

$$\vec{X} \times \vec{Y} = |\vec{X}|.|\vec{Y}|sin0^\circ$$

$$\vec{X} \times \vec{Y} = |\vec{X}|.|\vec{Y}| \times 0$$

Hence, the cross product of the parallel vectors becomes

$\vec{X} \times \vec{Y} = 0$, which is a unit vector.

2. Answer

i. (a) Metre (m)

ii. (c) Mass

iii. **Radius of gyration**: The radius of gyration of a body about an axis may be defined as the distance from the axis of a mass point whose mass is equal to the mass of the whole body and whose moment of inertia is equal to the moment of inertia of the body about the axis.

iv. **Theorem of perpendicular axes**
It states that the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body. If we consider a planar body, an axis perpendicular to the body through a point O is taken as the z-axis. Two mutually perpendicular axes lying in the plane of the body and concurrent with z-axis, i.e., passing through O, are taken as the x and y-axes. The theorem states that
$I_z = I_x + I_y$

v. The moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes. z and z' are two parallel axes, separated by a distance a. The z-axis passes through the centre of mass O of the rigid body. Then according to the theorem of parallel axes
$I_{z'} = I_z + Ma^2$
Where I_z and I_z' are the moments of inertia of the body about the z and z¢ axes respectively, M is the total mass of the body and a is the perpendicular distance between the two parallel axes.